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J. Phys. A: Math. Gen. 13 (1980) 3769–3772. Printed in Great Britain

Efficiency of quasispherical black hole accretion

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Received 15 April 1980

Abstract. The maximum efficiency for idealised accretion is considered, and previous remarks justified.

It has been remarked that the maximum possible efficiency obtainable from the most idealised configuration for quasispherical accretion onto a Schwarzschild black hole is just over 20% (Rees 1977). We give the reasoning behind this remark, and consider the actual maximum.

The idealisation we consider is the collision of two oppositely directed matter streams, where any velocity of the centre of mass is purely radial. All the kinetic energy perpendicular to this velocity is considered to be dissipated totally without further interaction, being radiated uniformly over the sky of the local rest frame. The notation, and all other references, are taken from Misner *et al* (1973, hereafter referred to as MTW, chapter 25). There are four different effects to consider: (i) the energy available locally, (ii) the red-shift factor for the radiation which escapes back out, (iii) the fraction of the radiation captured by the black hole itself, and (iv) relativistic beaming, if the residual velocity (the centre-of-mass velocity of the frame in which the radiation pattern is uniform) is non-zero.

For the first we have that the local energy per unit mass is given by

$$\tilde{E}_{\text{local}} = \tilde{E}\left(1-\frac{2M}{r}\right)^{-1/2} = (p_\phi p^\phi + p_r p^r + \mu^2)^{1/2}/\mu \tag{1}$$

where μ is the rest mass of the particle (or rest mass density of a stream) and p_ϕ, p_r are the momentum components in the local frame. Thus, the available energy is

$$\tilde{E}_{\text{local}} - \left(\frac{p_r p^r + \mu^2}{\mu^2}\right)^{1/2}. \tag{2}$$

Since the radial velocity is

$$v = p^r/p^0 = (1-\tilde{V}^2/\tilde{E}^2)^{1/2} \tag{3}$$

and the red-shift term (factor (ii)) is simply $[1-(2M/r)]^{1/2} = p^0 E$, we obtain a 'raw' efficiency by dividing throughout by the energy per unit mass at infinity, $\tilde{E}$, as

$$\epsilon_1 = 1 - \left(v^2\tilde{E}^2 + 1 - \frac{2M}{r}\right)^{1/2}\tilde{E}^{-1}. \tag{4}$$

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The 'potential barrier' to geodesic motion in the Schwarzschild field, $\tilde{V}$, is given by

$$\tilde{V}^2 = \left(1 - \frac{2M}{r}\right)\left(1 + \frac{\tilde{L}^2}{r^2}\right) \tag{5}$$

where $\tilde{L}$ is the angular momentum per unit mass.

We will not consider any collisions closer to the hole than $r = 3M$, because of the large capture cross section of the hole. Above $r = 3M$, all radiation emitted in a cone of semi-angle δ_m about the inward direction will be absorbed, where

$$\sin \delta_m = 3\frac{\sqrt{3}M}{r}\left(1 - \frac{2M}{r}\right)^{1/2} \tag{6}$$

(MTW box 25.7). If the distribution of the photons is isotropic in a frame travelling with velocity v with respect to the local rest frame, then the observed distribution is

$$n(\theta)\,d\theta \propto \frac{\sin\theta}{\gamma^2(1 - v\cos\theta)^2}\,d\theta \tag{7}$$

where θ is the angle with the forward v direction, and $\gamma^2 = (1 - v^2)^{-1}$. Integrating, the fraction beamed into a cone of semi-angle δ about the forward direction is

$$F(v, \delta) = \frac{(1 + v)(1 - \cos\delta)}{2(1 - v\cos\delta)}. \tag{8}$$

The possible collisions are between two infalling streams, two outflowing streams, or one of each. In the first two cases, the residual velocity is given by equation (3), and in the last it is zero. However, the latter two cases are only possible if $\tilde{E}$ is smaller than the maximum of $\tilde{V}$, which depends on $\tilde{L}$. Then there is a minimum radius of interaction, r_{min}, which occurs where $\tilde{V} = \tilde{E}$ and is one of the roots of the cubic

$$(1 - \tilde{E}^2)r^3 - 2Mr^2 + \tilde{L}^2 r - 2M\tilde{L}^2 = 0. \tag{9}$$

(See the discussion of the potential barrier in MTW.)

The maximum efficiency is due to competition between the capture cross section, which is smaller further away and for outward beaming, and the 'raw' efficiency ϵ_1 (equation (4)), which is larger nearer the hole. The total efficiency is given by combining (4), (6), (8) and (9):

$$\begin{aligned} \epsilon_I &= F(-v, \pi - \delta_m)\,\epsilon_1(-v) && \text{Infall, } r \geq 3M \\ \epsilon_O &= F(v, \pi - \delta_m)\,\epsilon_1(v) && \text{Outflow, } r \geq r_{min}(\tilde{E}, \tilde{L}) \\ \epsilon_H &= F(0, \pi - \delta_m)\epsilon_1(0) && \text{Head-on.} \end{aligned} \tag{10}$$

Note that, where r_{min} exists, $\epsilon_I = \epsilon_O = \epsilon_H$ at $r = r_{min}$, as expected because all motion is tangential there.

The figure commonly quoted corresponds to parabolic infall, $\tilde{E} = 1$, at the smallest possible r_{min}, which is $4M$ and occurs for $\tilde{L} = 4M$. This is an efficiency of 0·2043, and is the maximum possible for $\tilde{E} = 1$. Figure 1 shows ϵ_I, ϵ_O and ϵ_H for $r \geq 4M$, for $\tilde{E} = 1$, $\tilde{L} = 4M$. For $\tilde{L} > 4M$, r_{min} increases and ϵ_1 decreases faster than F can compensate. For $\tilde{L} < 4M$, there is no r_{min}, but the hole captures sufficiently more to overcome the increase in ϵ_1. (Note that ϵ_I is slightly larger—a few parts in 10^5—for r just less than r_{min}, which is (strictly) not allowed, but just possible because $r = 4M$ is an unstable circular orbit.)

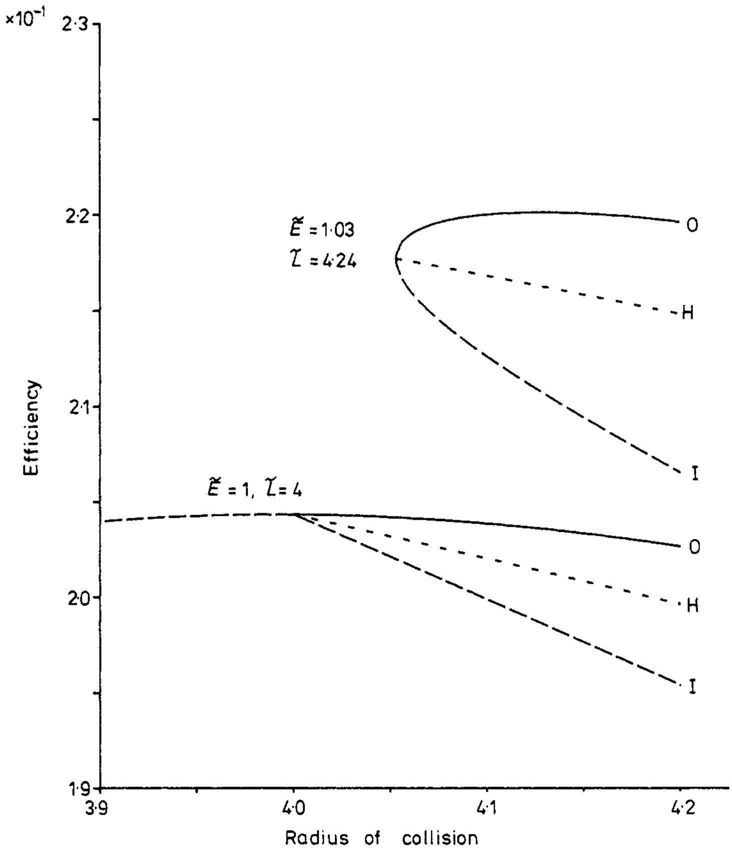

Figure 1. The accretion efficiency for two different combinations of $\tilde{E}$ and $\tilde{L}$, corresponding to the three cases of infalling (I), out-flowing (O), and head-on (H) collisions (see equations (10)).

Although parabolic accretion is probably the most likely physically, because it corresponds to zero velocity at infinity, other values of $\tilde{E}$ are also of interest. Smaller energies, $\tilde{E} < 1$, correspond to matter initially at rest rather closer than infinity. More energetic accretion, $\tilde{E} > 1$, occurs with unbound matter, perhaps coming from explosive processes. Small variations around $\tilde{E} = 1$ change the resultant efficiencies, and it is possible to add a per cent or so. Thus for $\tilde{E} = 1{\cdot}02$, there is an efficiency of 0·2169 for outward-beamed collisions at $r = 4M$, with $\tilde{L} = 4{\cdot}156M$. For $\tilde{E} = 1{\cdot}03$, there is a value of 0·2201 for outward-beamed collisions with $\tilde{L} = 4{\cdot}24M$, at $r = 4{\cdot}1M$. It is generally true that ϵ_{I} and ϵ_{H} decrease as r increases beyond $r_{\min}$, whilst ϵ_{O} peaks somewhat outside of $r_{\min}$. Figure 1 also shows some curves with $\tilde{E} > 1$.

Thus we see that idealised dissipative quasispherical accretion can reach an efficiency of 22%, and why this is so.

Acknowledgments

I am grateful to Martin Rees for posing this simple problem during his quasar studies.

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